Uniform Acceleration Radiation and the Equivalence Principle

Timothy Bates

Texas A&M University

Defining Energy and Radiation

We consider a uniformly accelerating source in a massless scalar field, whose motion is depicted in the figure below, and which is given by $\phi(x) = \frac{q}{4\pi R}\theta(z+t)$, worked out in [4]. Our goal is to compute radiation via the Poynting flux of a scalar field with respect to an observer's proper timelike Killing vector.

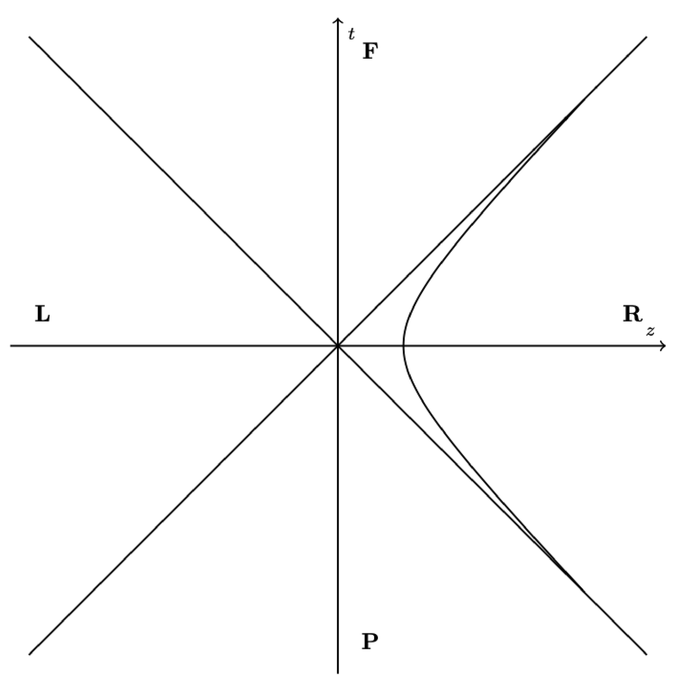


Fig. 1: Hyperbolic Motion with 4 labeled regions

We define energy as the conserved quantity associated to a timelike Killing vector field. Killing vectors fields correspond to the vanishing of the Lie derivative of the metric; in other words, the metric is left unchanged under infinitesimal translations by a Killing vector. These generators of symmetries are associated via Noether's theorem to a conserved quantity. For inertial observers, the time translation vector field ∂_t is a Killing vector (as it is independent of the Minkowski metric) that corresponds to the conserved quantity of energy. In Rindler space, the notion of Energy is different. Rindler energy is the conserved quantity associated with the Killing field generated by the timelike Killing vector

$$\partial_\lambda = z\partial_t + t\partial_z = \frac{1}{2}[v\partial_v - u\partial_u] \tag{1}$$

where λ is Rindler time. Notice that this is also the Lorentz boost Killing vector in Minkowski space.

Now we define Poynting flux. Contracting a timelike Killing vector with the stress energy tensor captures energy flux. To quantify the flow, we contract with a spacelike vector normal to a constant time surface. This is naturally interpreted as energy flux per unit area in the outward normal direction. Integrating across all spatial angles yields power, quantifying the energy flow out of a region:

$$S = -T_{\mu\nu}\xi^\mu\hat{n}^\nu \tag{2}$$

where ξ^μ is the Killing vector, $\hat{n}^\nu$ is a unit normal vector to the surface of integration, and $T_{\mu\nu}$ is the stress tensor.

Scalar Field Radiation

Power radiated for Minkowski and Rindler observers for a uniformly accelerating source.

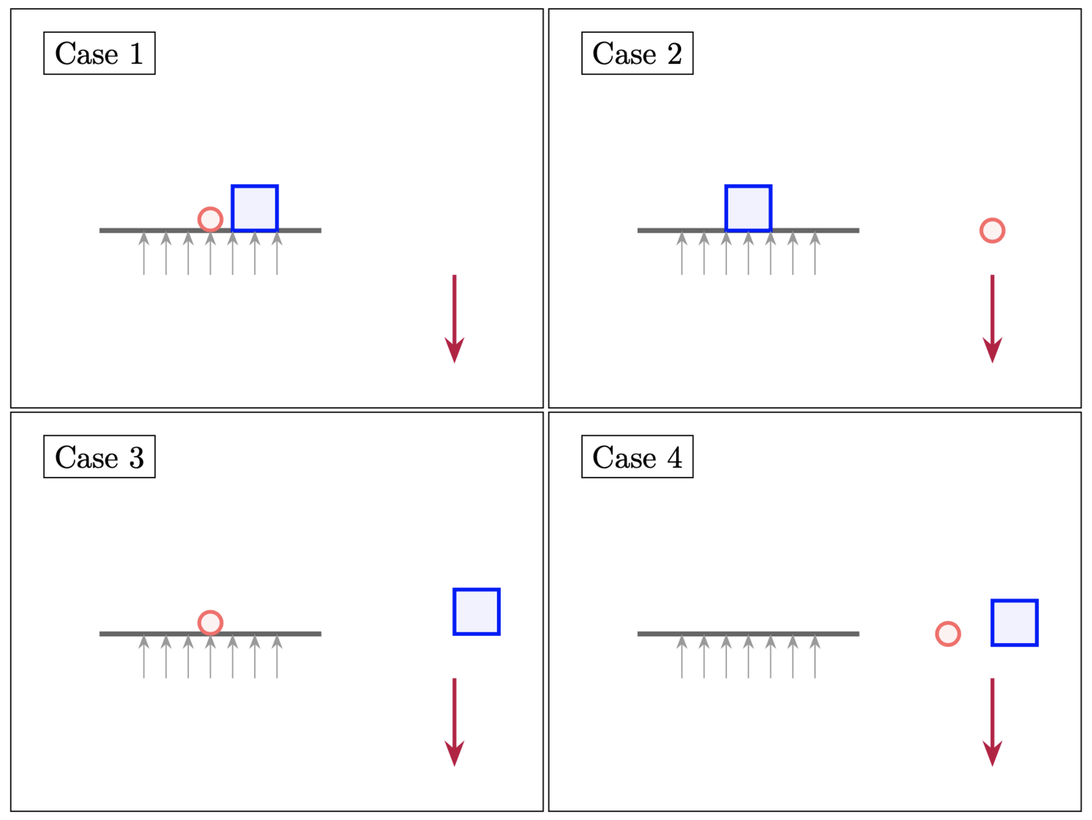
$$\mathscr{R}_M = \frac{q^2a^2}{12\pi}, \quad \mathscr{R}_R = 0$$

Equivalence Principle Paradox

The equivalence principle glues together special relativity and gravity. There are different statements for the 'equivalence principle' which are not necessarily equivalent. Relevant to us is the qualitative equivalence principle. It captures the relevant physics for uniform acceleration, namely that

> uniformly accelerating frames are qualitatively equivalent to static frames in gravitational fields.

The paradox comes from considering the following thought experiment [1][2][3] in which we have a source and an observer in four cases. In which cases does the observer detect radiation?



The QEP prescribes us to treat 1 & 4 similarly, and 2 & 3 similarly, in answering which cases observe radiation. To verify this computationally, cases 1 and 2 have supported observers, and cases 3 and 4 have free falling observers. By the qualitative equivalence principle it follows that for a supported frame in a uniform gravitational field, given by the Rindler metric

$$ds^2 = -g^2Z^2d\lambda^2 + dZ^2 + dx^2 + dy^2 \tag{3}$$

, scalar radiation of supported observers is the physics of Rindler observers. This treatment was done for electromagnetic fields in [3]. Thus we can use the Minkowski and Rindler Poynting flux vectors to compute radiation. We expect that cases 1 & 4 report no radiation, while cases 2 & 3 report radiation.

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References

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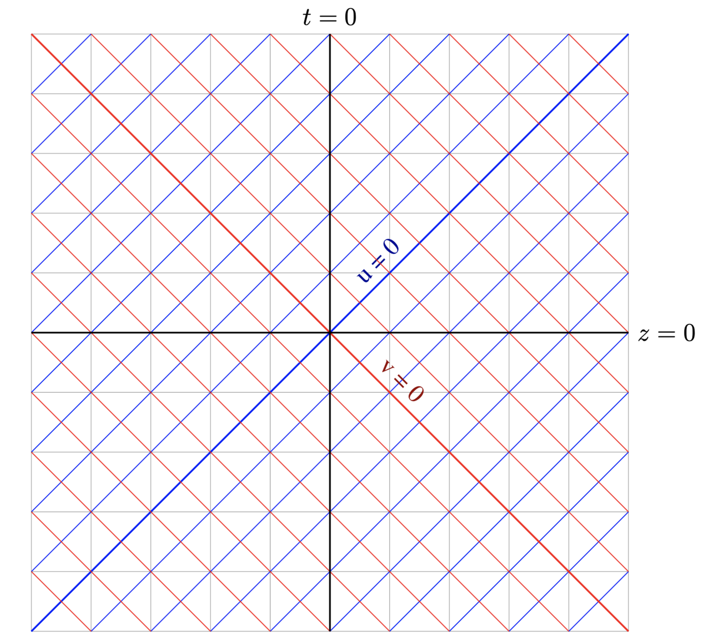
Divergence of the Stress Tensor

Working with the divergencelessness of the stress tensor, it becomes convenient to consider a coordinate system which we call 'null coordinates'. Let $u = t - z$, $v = t + z$, $x = x$, $y = y$. u and v are null lines which on a spacetime diagram correspond to the rightward and leftward lightrays.



We expect a stress tensor of a field which satisfies energy conservation to have a divergence which vanishes; $\nabla_\mu T^{\mu\nu} = 0$. The scalar stress tensor $T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi$, is determined to vanish on the past horizon, and in $R \cup F$. To help understand how this divergence holds, we can study the flow of the stress tensor contracted with the Minkowski Killing vector: $S^\alpha = -g^{\alpha\nu}T_{\mu\nu}\xi^\mu$.

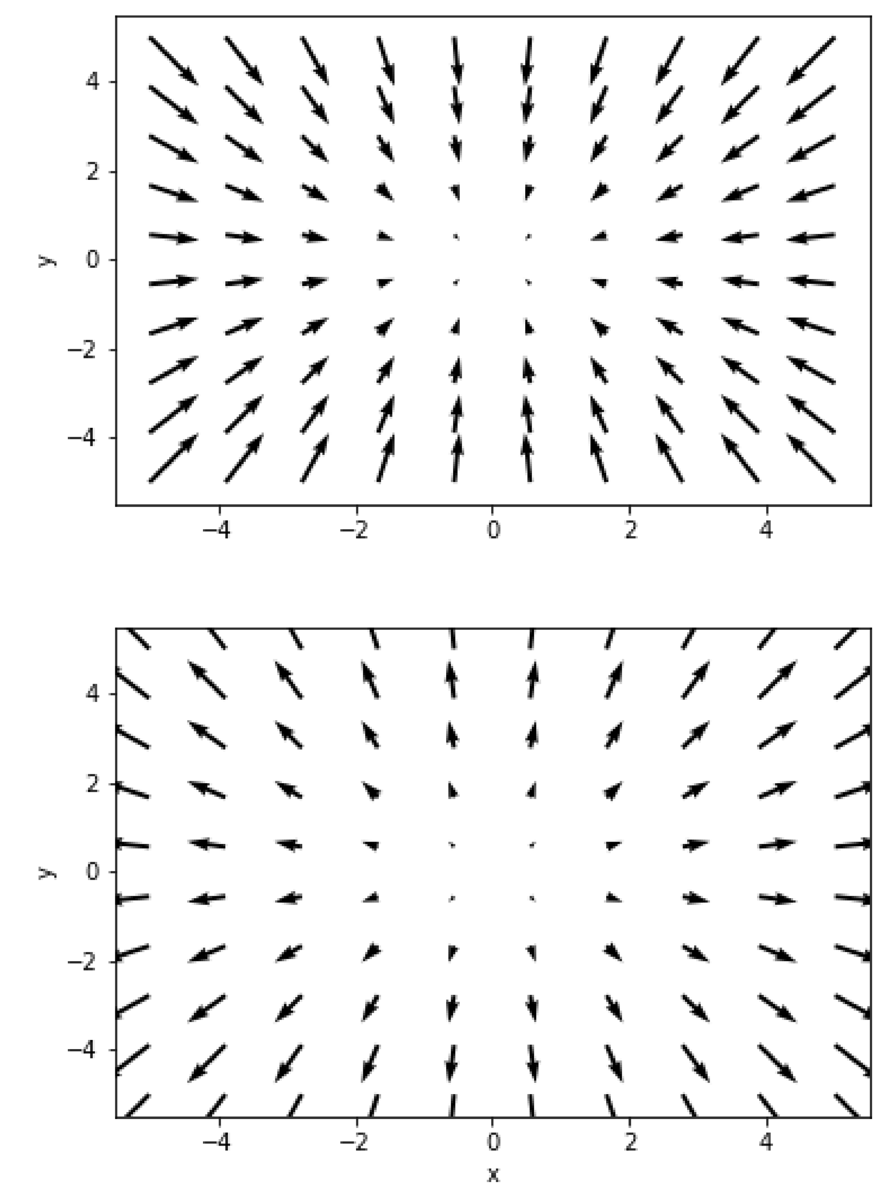


Fig. 5: Top plot $|u| > v$, bottom plot is for $|u| < v$

In the R region, for $|u| < v$, the flux flows inward to the plane of the source in the transverse directions. When we get close to the future horizon (u = 0), the flux starts moving outward to the transverse directions. When $|u| < |v|$, we have a inward flux, and when $|v| > |u|$, we have outward flux. This is a result of the fact that at these points, the sum of components inside the S^x and S^y expressions change sign. How does this look on the z–t plane? $-u = v$ in the R region is the line $t = 0$, so when t is negative, we have inward flux. When t is positive, we have outward flux.