Ginzburg study and summary

This is merely meant to be a summary together with some personal notes and observations. Much is direct wording from the original text and so should not be reproduced citing me.

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Preface This document is meant to summarize chapter 3 of Vitaly Ginzburg's Theoretical Physics and Astrophysics [3]. The original text was fairly difficult to obtain and was also not in an electronic format and thus motivated creating this document to summarize the important points relevant to us in an electronic form.

1 Chapter 3: Uniformly Accelerated Charge

1.1 Exposition of Uniformly Accelerated charges

(In this text the $\wedge$ symbol denotes cross product. Bold symbols are vectors.)

Ginzburg introduces the uniformly accelerated charge as a long standing problem. The problem emerges since when the acceleration is uniform $\ddot{v}$ is zero, thus the radiation force $f = (2e^2/3c^3)\ddot{v}$ vanishes but the Larmor formula (1) predicts that it does radiate. The issue then is how can a charge radiate when its radiation reaction force is zero?

$$\mathscr{P} = \frac{2e^2}{3c^3}\dot{\mathbf{v}}^2 \quad \text{non-relativistic Larmor formula} \tag{1}$$

He briefly discusses periodic motion (noting that work done by radiation force is not always equal to $\mathscr{P}$ and that in periodic motion the total energy is conserved on average over time) and then goes into a more general discussion of acceleration radiation (not purely discussing the case of the periodic motion). Given a charge e moving along some trajectory, the electromagnetic field is determined from the Lienard-Wiechert potentials.

$$\mathbf{E} = \frac{e(1-\frac{v^2}{c^2})}{[R-(\mathbf{v}\cdot\mathbf{R})/c]^3}\left(\mathbf{R}-\frac{\mathbf{v}R}{c}\right) + \frac{e}{c^2[R-(\mathbf{v}\cdot\mathbf{R})/c]^3}\left[\mathbf{R}\wedge\left[\{\mathbf{R}-\frac{\mathbf{v}R}{c}\}\wedge\dot{\mathbf{v}}\right]\right] \tag{2}$$

$$\mathbf{H} = \frac{1}{R}[\mathbf{R}\wedge\mathbf{E}] \tag{3}$$

Fields are taken at 'observation point' at time t while the RHS of the equation quantities $\mathbf{R}$, $\mathbf{v}$, and $\dot{\mathbf{v}}$ refer to emission(retarded) time $t' = t - R(t')/c)$ where the vector $\mathbf{R}$ points from the charge e at t' to the observation point. The velocity of charge is $v(t') = -\partial R(t')/\partial t'$ and $\dot{v} = \partial v/\partial t'$. We have $r(t) = r(t') + R(t')$.

The first term of $\mathbf{E}$ corresponds to field of charge moving with velocity v which decreases by $1/R^2$ and second term decreases by $1/R$ which becomes the dominant term for $R >> c^2(1-v^2/c^2)/\dot{v}$ and this term describes a transverse field of an electromagnetic wave. "If charge produces such a wave field one says that it radiates." "it[radiation] also needs a more precise definition. Indeed, we can consider the wave field of a charge, which decreases as 1/R, only in the wave zone where only a single such field can exist in practice." Later the claim is made that "it is advisable to understand the statement 'the charge radiates' in a wider sense, namely, as the presence of a wave field, independent of whether or not another part of the field is present. We must also emphasize that when we measure the fields E and H at time t we can only reach a conclusion about the state of the electron at an earlier time." This is all to emphasize the necessary precision in defining radiation.

Now consider the (now relativistic) field of a single charge. The energy flux through a closed surface surrounding the charge must be non-vanishing when there is radiation present.

$$dW_s = \frac{c}{4\pi}\left([\mathbf{E}\wedge\mathbf{H}]\cdot s\right)R^2 d\Omega dt = \frac{e^2}{4\pi c^3}\frac{[\mathbf{s}\wedge[(\mathbf{s}-\mathbf{v}/c)\wedge\dot{\mathbf{v}}]]^2}{[1-(\mathbf{s}\cdot\mathbf{v})/c]^6}d\Omega dt \tag{4}$$

In general this is only valid in the wave zone. The total energy emitted per unit time t' is (by integrating over solid angle Ω)

$$\mathscr{P} = \frac{dW}{dt'} = \frac{e}{4c}\int\frac{[\mathbf{s}\wedge[(\mathbf{s}-\mathbf{v}/c)\wedge\dot{\mathbf{v}}]]^2}{[1-(\mathbf{s}\cdot\mathbf{v})/c]^6}d\Omega$$

$$= \frac{2e^2}{3c^3}\frac{\dot{\mathbf{v}}^2-(\mathbf{v}\cdot\dot{\mathbf{v}})^2/c^2}{(1-v^2/c^2)^3} = -\frac{2e^2c}{3}w^i w_i$$

where w^i is the acceleration four vector of the particle. Since this equation is Lorentz invariant it can be evaluated in any inertial frame. If the velocity of the charge at radiation time t is zero then

$$\mathscr{P} = \frac{2e^2}{3c^3}\dot{v}^2$$

For non-relativistic uniformly accelerated motion we have $\dot{\mathbf{v}}$ = a constant. A motion in which acceleration is constant in the moving frame of reference where the particle velocity is zero, is called a relativistic uniformly accelerated motion. In this frame we always have $\ddot{\mathbf{v}}=0$. This can be written in the form

$$\frac{dw^i}{ds} + \alpha u^i = 0$$

Where u^i is the velocity four vector and where α is a constant; when $\mathbf{v}=0$, this indeed becomes the equation $\ddot{\mathbf{v}}=0$. We have

$$\alpha = w^i w_i$$

We have that

$$\frac{dw^i}{ds} + w^k w_k u^i = 0$$

In three dimensional notation this is

$$\left(1-\frac{v^2}{c^2}\right)\ddot{\mathbf{v}} + \frac{3}{c^2}(\mathbf{v}\cdot\dot{\mathbf{v}})\dot{\mathbf{v}} = 0$$

Of interesting note here is that this is a derivation of the equation of uniform accelerated motion identical to expression (2.1) in Fulton and Rohrlich [1]

The function z(t) is a hyperbola which is relativistic uniformly accelerated rectilinear motion. The motion is also hyperbolic in the corresponding gravitational field. It is clear that in the relativistic and non-relativistic uniformly accelerated motion the charge radiates and that $\mathscr{P} = dW/dt' = (2e^2/3c^3)w^2$.

The form of equation of motion of a charge had the (2.1) form (in Ginzburg) which was

$$m\dot{\mathbf{v}} = F_0 + \frac{2e^2}{3c^3}\ddot{\mathbf{v}}$$

Relativistic generalization where external force assumed to be the Lorentz force

$$mc\frac{du^i}{ds} = \frac{e}{c}F_0^{ik}u_k + \frac{2e^2}{3c}\left(\frac{d^2u^i}{ds^2} + u^i\frac{du^k}{ds}\frac{du_k}{ds}\right) \tag{5}$$

1.2 Three points lacking clarity

While the first section outlined the background and exposition of the problem, the next section discusses the paradoxes and attempts to address and resolve them.

(1) The first lack of clarity: The presence of radiation notwithstanding the vanishing of the radiation reaction force.

(2) The second lack of clarity: The case of a uniformly accelerated charge is connected with the application of the equivalence principle for the motion of a charge in the uniform gravitational field. He does not directly address this in this book but cites Ginzburg 1970c which is an article in the Soviet Physics Uspekhi journal.[2]

(3) The third lack of clarity: The third lack of clarity arises when one attempts to describe for all t and z the field of a charge which moves with a uniformly accelerated motion for $-\infty < t < \infty$).

Addressing the points He addresses the third lack of clarity first. He quotes Leibovitz and Peres who say that they are 'led to the conclusion that the Maxwell equations are incompatible with the existence of a single charge uniformly accelerated at all times'[4]. Ginzburg agrees arguing that since the total energy emitted is infinite and the kinetic energy is also infinite that this would be an ill-posed problem. He then restricts to uniform acceleration in finite time giving as an example the motion in a uniform and constant electric field (condenser). The charge accelerates for time $t' \in (t'_1, t'_2)$ and has constant velocity elsewhere. He remarks that in a condenser the motion is hyperbolic only when the field vector and particle velocity are parallel. He remarks that in the finite case "one can find the solution for the field in the form of retarded potentials."

As noted previously the wave zone field decreases as $1/R$ for $R >> c^2(1 - v^2/c^2)/\dot{v}$ however this zone does not exist at all for hyperbolic motion at a fixed observation time t. The equations of a particle performing hyperbolic motion yield $\dot{v} = c^3/w^2[(c^2/w^2) + t^2]^{3/2}$ which is approximately c^3/w^2t^3 and since we are observing from t, the motion from a time t' we have $\dot{v} \approx c^3/w^2t'^3$ and $1 - \frac{v^2}{c^2} \approx c^2/w^2t'^2$ so $(c^2/\dot{v})(1 - \frac{v^2}{c^2}) \approx ct'$ and thus since $R = c(t - t')$ goes to infinity as t' goes to minus infinity we have that $R >> ct'$ is not met since the magnitudes of both increase proportionally. The next sentence confuses me. He states "Hereby the well known arbitrariness of the concept of the energy emitted by the charge is clear: one must state about what value of t or t' one is speaking." I believe the sentence is merely saying one needs to be careful in doing calculations when choosing t or t'.

The problem is nevertheless well defined for a motion which has uniform acceleration during a finite time interval. One can say that during the time of uniform acceleration that no radiation force was acting upon the charge and at the same time the charge radiated: the flux of energy through a sphere of radius R(t') at time $t = t' + R/c$ was nonzero.

He now addresses the paradox of presence of radiation in absence of radiation force (this is the first lack of clarity). To determine energy emitted by the charge or radiation intensity at a given surface one evaluates the Poynting vector $\mathbf{S} = c[\mathbf{E} \wedge \mathbf{H}]/4\pi]$ far from charge and finds flux of this vector through a closed surface. (This was how the Larmor formula was derived). These results are only valid in a vacuum. For a medium we get in general completely different results. A uniformly moving charge can radiate in a medium in for example: Cherenkov or transition radiation (chap 6, 7). One can calculate energy losses of a charge (emitted energy) in two different ways: through determining the time-derivative of the energy in the field, $(d/dt)\int[(\mathbf{E} \cdot \mathbf{D}) + H^2]d^3r/8\pi$ or by finding work done, $e(\mathbf{v} \cdot \mathbf{E}') = (\mathbf{v} \cdot \mathbf{f})$ by the charge against the field produced by the charge itself. One calculates the work done by the radiation reaction force $\mathbf{f}$. For several cases all three methods lead to same result. (for example Cherenkov radiation). However, in general the total energy flux, the change in the field energy, and the work done by the radiation force are not equal to one another.

The paradox in question is connected with the illegitimate identification of the energy flux with the work done by the radiation force.

$$\frac{d}{dt}\left(\frac{E^2+H^2}{8\pi}\right) = -(\mathbf{j}\cdot\mathbf{E}) - \nabla\cdot\mathbf{S}, \quad S = \frac{c}{4\pi}[\mathbf{E}\wedge\mathbf{H}] \text{ Poynting's theorem} \tag{6}$$

He considers the motion of single point charge in vacuum $\mathbf{j} = e\mathbf{v}\delta(\mathbf{r}-\mathbf{r}_e(t))$

Integrates over volume and surface around the charge

$$\frac{d\mathscr{H}_{em}}{dt} = -e(\mathbf{v}\cdot\mathbf{E}) - \oint S_n d^2\sigma, \mathscr{H}_{em} = \int \frac{E^2+H^2}{8\pi} d^3r$$

Obtain from equations of motion,

$$\frac{d}{dt}\left[\frac{m\mathbf{v}}{\sqrt{1-(v^2/c^2)}}\right] = e\left(E_0 + \frac{1}{c}[\mathbf{v}\wedge\mathbf{H}_0]\right) + f,$$

$$\mathbf{f} = \frac{2e}{3c[1-(v^2/c^2)]}\left(\ddot{\mathbf{v}} + \dot{\mathbf{v}}\frac{3(\mathbf{v}\cdot\dot{\mathbf{v}})}{c^2[1-v^2/c^2]} + \frac{\mathbf{v}}{c^2[1-v^2/c^2]}\left((\mathbf{v}\cdot\dot{\mathbf{v}}) + \frac{3(\mathbf{v}\cdot\dot{\mathbf{v}})^2}{c^2[1-v^2/c^2]}\right)\right)$$

this expression by dotting with the velocity three vector $\mathbf{v}$,

$$\frac{d\mathscr{E}}{dt} = e(\mathbf{v}\cdot\mathbf{E}_0) + (\mathbf{v}\cdot\mathbf{f}), \quad \text{where} \quad \mathscr{E} \equiv \mathscr{H} = \frac{mc^2}{\sqrt{1-v^2/c^2}}$$

Next one adds $d\mathscr{H}_{em}/dt$ and $d\mathscr{E}/dt$ to obtain

$$\frac{d(\mathscr{H}_{em}+\mathscr{E})}{dt} = e(\mathbf{v}\cdot\mathbf{E}_0) - e(\mathbf{v}\cdot\mathbf{E}) - \oint S_n d^2\sigma + \mathbf{v}\cdot\mathbf{f}$$

$$= e(\mathbf{v}\cdot\mathbf{E}_0 - \mathbf{v}\cdot(\mathbf{E}_0+\mathbf{E}')) - \oint S_n d^2\sigma + \mathbf{v}\cdot\mathbf{f}$$

Since $\mathbf{E}'$ is the field of the charge itself and $e\mathbf{E}' = \mathbf{f}$

$$= -\mathbf{v}\cdot\mathbf{f} + \mathbf{v}\cdot\mathbf{f} - \oint S_n d^2\sigma$$

which reduces to this expression

$$\frac{d(\mathscr{H}_{em}+\mathscr{E})}{dt} = -\oint S_n d^2\sigma$$

Thus the time derivative of the total energy of (the field energy $\mathscr{H}_{em}$ and the charge $\mathscr{E}$ is the minus integral of the Poynting vector over a surface surrounding the charge.

If one assumes (for simplification) that the charge is accelerated by some external non-electromagnetic field: $\mathbf{E} = \mathbf{E}'$ thus

$$\frac{d\mathscr{H}_{em}}{dt} = -e(\mathbf{v}\cdot\mathbf{E}) - \oint S_n d^2\sigma$$

$$= -(\mathbf{v}\cdot(e\mathbf{E}')) - \oint S_n d^2\sigma$$

$$= -\mathbf{v}\cdot\mathbf{f} - \oint S_n d^2\sigma$$

Thus the conservation law becomes

$$\frac{d\mathscr{H}_{em}}{dt} = -(\mathbf{v}\cdot\mathbf{f}) - \oint S_n d^2\sigma \tag{7}$$

Where $\mathscr{H}_{em}$ is energy of the field of the charge (assuming all other electromagnetic fields are absent). With this it is clear that the work done by radiation force $\mathbf{v}\cdot\mathbf{f}$, change in field energy $d\mathscr{H}_{em}/dt$, and total energy flux $\int S_n d^2\sigma$ are connected but not equal in general. Consider a stationary motion where $d\mathscr{H}_{em}/dt = 0$: $-(\mathbf{v}\cdot\mathbf{f}) = \oint S_n d^2\sigma$. By moving surface σ to infinity when $\oint S_n d^2\sigma = 0$: we have $d\mathscr{H}_{em}/dt = -(\mathbf{v}\cdot\mathbf{f})$. This is why we can determine the energy losses of a particle $(\mathbf{v}\cdot\mathbf{f})$ in stationary regime by evaluating $\oint S_n d^2\sigma$ or $d\mathscr{H}_{em}/dt$.

Considering the hyperbolic motion again in the relativistic case we can write

$$\frac{d}{dt'}\left[\frac{mc^2}{\sqrt{1-v^2/c^2}}\right] = e(\mathbf{v}\cdot\mathbf{E}_0) + \frac{2e^2}{3}\left(\frac{dw^0}{ds} + cw^i w_i\right)$$

We can rewrite as

$$\frac{d}{dt'}\left[\frac{mc^2}{\sqrt{1-v^2/c^2}}\right] = e(\mathbf{v}\cdot\mathbf{E}_0) + (\mathbf{v}\cdot\mathbf{f}) = e(\mathbf{v}\cdot\mathbf{E}_0) + \frac{2e^2}{3}\frac{dw^0}{dt'} - \mathscr{P}$$

Thus

$$(\mathbf{v}\cdot\mathbf{f}) = \frac{2e^2}{3}\frac{dw^0}{dt'} - \mathscr{P}$$

(Note: $2e^2w^0/3$ is the Schott term.) One then has that for the condenser the charge radiates at a rate $\mathscr{P} = \frac{2e^2}{3c^3}w^2$ for $t' \in (t'_1, t'_2)$ which yields a total radiated power of $\frac{2e^2}{3c^3}w^2(t'_2 - t'_1)$. One also has that the radiation force does not act for $t' \in (t'_1, t'_2)$ nor for $t' < t'_1$ nor $t' > t'_2$ but does act at exactly the points t'_1 and t'_2 and that the total radiated power is given by

$$\int_{t'_1}^{t'_1} (\mathbf{v}\cdot\mathbf{f})dt' = -\int_{t'_1}^{t'_2} \mathscr{P}dt' = -\frac{2e^2}{3c^3}w^2(t'_2 - t'_1)$$

which is exactly equal to the energy emitted. Ginzburg states that it is clear that the vanishing of the radiation force during uniformly accelerated motion is in no sense paradoxical. He states that “the energy flux through a surface is directly determined by the field near the surface, and not by the field on the trajectory of the charge which is inside the surface.”

Next he addresses Fulton and Rohrlich where he states that the conservation law is not used but rather the concept of an acceleration energy is introduced $Q = \frac{2e^2w^0}{3}$. This is expression 4.8 in Fulton and Rohrlich[1] and is also the Schott term.

It is clear that $(\mathbf{v}\cdot\mathbf{f}) = dQ/dt' - \mathscr{P}$ and that

$$\frac{d\mathscr{E}}{dt'} - (\mathbf{v}\cdot\mathbf{f}) = \frac{d\mathscr{E}}{dt'} - \left(\frac{dQ}{dt'} - \mathscr{P}\right) = e(\mathbf{v}\cdot\mathbf{E}_0)$$

The quantity Q is sometimes interpreted as part of the ‘internal energy of a charged particle’ or sometimes assumed to be part of the energy of the field which immediately surrounds the particle, without contributing to its electromagnetic mass. From these points of view we may assume, when the radiation force vanishes, that the emitted energy $\mathscr{P}$ derives from either the acceleration energy Q or from the internal energy $\mathscr{E} - Q$. He states that if we assume that Q is part of the field energy, the radiation energy $\mathscr{P}$ derived from field energy is correct because $\mathscr{P} = dW/dt'$ is flux of field energy through a surface surrounding the charge. He then goes on to claim that introduction of ‘acceleration energy’ or ‘internal energy’ of charge does not add to any understanding and instead complicates the problem. He states that it may be natural to write work as a sum of two terms but no need to give these terms any new meaning.

My understanding of Rohrlich’s point of view is that one may either accept the equations of motion and then accept as a consequence that the Schott term becomes part of the internal energy of the particle or choose not to accept the equation of motion. Rohrlich in his paper also acknowledges that the Schott energy will be part of the surrounding field which moves with the charge. Ginzburg on the other hand uses the same equations of motion and rejects the notion that this Schott term is part of the internal energy of the particle. Ginzburg does accept it as part of the field around the particle. Despite this philosophical disagreement they both end up with the ‘Schott term’ and numerically they agree.

References

[1] Thomas Fulton and Fritz Rohrlich. Classical radiation from a uniformly accelerated charge. *Annals of Physics*, 9:499–517, 1960.

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[3] V.L. Ginzburg. Chapter iii - uniformly accelerated charge. In V.L. Ginzburg, editor, *Theoretical Physics and Astrophysics*, International Series on Nuclear Energy, pages 37–51. Pergamon, Amsterdam, 1979.

[4] Clement Leibovitz and Asher Peres. Energy balance of uniformly accelerated charge. *Annals of Physics*, 25(3):400–404, 1963.